AGMA Technical Paper

A New Statistical Model for Predicting Tooth Engagement and Load Sharing in Involute Splines

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A New Statistical Model for Predicting Tooth Engagement and Load Sharing in Involute Splines

Janene Silvers, Carl D. Sorensen and Kenneth W. Chase, Brigham Young University

[The statements and opinions contained herein are those of the author and should not be construed as an official action or opinion of the American Gear Manufacturers Association.]

Abstract

Load-sharing among the teeth of involute splines is little understood. Designers typically assume only a fraction of the teeth are engaged and distribute the load uniformly over the assumed number of engaged teeth. This procedure can widely over- or underestimate tooth loads.

A new statistical model for involute spline tooth engagement has been developed and presented earlier, which takes into account the random variation of gear manufacturing processes. It predicts the number of teeth engaged and percent of load carried by each tooth pair. Tooth-to-tooth variations cause the clearance between each pair of mating teeth to vary randomly, resulting in a sequential, rather than simultaneous tooth engagement. The sequence begins with the tooth pair with the smallest clearance and proceeds to pick up additional teeth as the load is increased to the maximum applied load. The new model can predict the number of teeth in contact and the load share for each at any load increment.

This report presents an extension of the new sequential engagement model, which more completely predicts the variations in the engagement sequence for a set of spline assemblies. A statistical distribution is derived for each tooth in the sequence, along with its mean, standard deviation and skewness. Innovative techniques for determining the resulting statistical distributions are described. The results of an in-depth study are also presented, which verify the new statistical model. Monte Carlo Simulation of spline assemblies with random errors was performed and the results compared to the closed-form solution. Extremely close agreement was found. The new approach shows promise for providing keener insights into the performance of spline couplings and will serve as an effective tool in the design of power transmission systems.

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American Gear Manufacturers Association
500 Montgomery Street, Suite 350
Alexandria, Virginia, 22314

October 2010

ISBN: 978-1-55589-982-0
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Introduction
Splined shafts are preferred over keyed shafts for transmitting heavy torque in industrial and automotive applications. The splined shaft and mating hub have matching sets of teeth over the full circumference, as shown in Figure 1. If the tooth loads were distributed uniformly around the circumference, each tooth would carry an equal share. However, due to manufacturing variations, the tooth clearance between each pair of teeth varies, so the teeth do not engage all at the same time. Thus, the load is not shared uniformly.

In practice, as the shaft is turned, the tooth pair with smallest clearance gap will make contact first and begin to carry the load. As the torque increases, the first tooth deflects enough for a second pair, with the next smaller clearance, to engage and begin to share the load. This process of sequential engagement continues with increasing load until the full load is applied. The full load is generally not sufficient to engage all of the teeth, so some teeth will not carry any load.

As a result of this sequential engagement, the first pair of teeth to engage will carry more of the load, causing the first tooth to be most likely to fail. Each tooth in succession will carry a smaller share. The motivation behind this research is to permit designers to accurately predict the tooth loading and avoid spline failure.

Previous work
Tooth engagement is driven by deflection: as the force increases and the engaged teeth deflect, clearance gaps between other tooth pairs close and additional tooth pairs engage. This deflection can be described using a strength of materials deflection model, as was done by DeCaires [1]. His model encompasses deflection due to shear, bending, and contact forces, and was verified by FEA. Figure 2 shows a force-deflection curve calculated using this model.

Each tooth is modeled as a spring, which acts in parallel with the other teeth. When multiple pairs of teeth are engaged, the stiffness, $K_i$, of each add together, which can be seen in the figure. At each data point, another pair of teeth engages, changing the slope incrementally.

Figure 1. External and internal spline teeth
The amount of load carried by a given tooth can be found by extending the slope of each segment of the graph, as shown, and then measuring the vertical distance between segments at the deflection value corresponding to the applied force. The first tooth to engage, hereafter referred to as Tooth 1, carries the largest load.

DeCaires’ model can be used to determine the percentage of the total load carried by Tooth 1. The percentage is compared to the number of teeth engaged in Figure 3. Tooth 1 always carries a larger percentage of the total load than any other teeth. Although the total load on Tooth 1 continues to increase, the percent of the total decreases due to load sharing by an increasing number of teeth.

**Models for tooth clearance variation**

Multiple sources of error are present in tooth manufacturing for both the internal and external teeth, so the resulting tooth clearance is a combination of several random variables. Therefore, a normal distribution of tooth clearances is a reasonable assumption. This distribution is shown in Figure 4. Note that more clearances are clustered near the middle of the distribution and spread out near the tails. The teeth are self-sorting in order of increasing clearance—teeth will engage in order, from the smallest clearance to the largest clearance, regardless of their location in the assembly.
Mapping model

One method of predicting the clearance variation is the mapping model, shown in Figure 5 for a 10-tooth spline. A uniform distribution is plotted on the y-axis and divided into 10 equal intervals. The center point of each interval is projected horizontally across to intersect the normal cumulative distribution function (CDF), then vertically down to the x-axis. The resulting distribution on the x-axis is normal. This model predicts the mean, or most likely, clearance values for the first tooth to engage, the second tooth to engage, and so on.

![Figure 4. Normal distribution of tooth clearances](image)

The horizontal axis has units of standard deviation. If the several process errors are known from inspection data, or estimated from previous experience, their standard deviations may be added by root-sum-squares to estimate the resultant clearance standard deviation. The horizontal axis may thus be scaled to the corresponding dimensional clearance.

Using the CDF to transform one distribution into another is an established procedure [2]. In this case, the center point of each increment, when mapped across and down to the horizontal axis, locates the most likely clearance for each tooth pair. In reality, the clearance of Tooth 1, Tooth 2, etc. is subject to random variation about the most likely value. This is an important aspect of tooth engagement, which is complex statistically. It is dealt with in the next section.

Probabilistic tooth engagement model

If two splined shafts are assembled to two hubs, the resulting clearances will be similar, but not exactly the same. Since the process variations are random, each assembly will have slightly different clearances, just as no two snowflakes are exactly alike. Furthermore, if a spline is disassembled, the shaft is rotated a couple of teeth, and reassembled, all the clearances are rearranged. This means that both the force-deflection curve and the load carried by
Tooth 1 vary for each spline assembly and for each alternate meshing of the same spline-hub pair. Consequently, neither can be defined for all cases, but they can be predicted by statistically modeling the expected clearance variation.

For a given spline assembly, the clearance for every tooth pair could be measured in theory, although this would be difficult in practice. As a large number of spline assemblies is measured, a histogram for the clearance of each tooth pair can be developed, and will approach a continuous distribution.

In this paper, a more complete statistical model to predict tooth clearances is developed and validated. This model is called the Probabilistic Tooth Engagement Model, or ProTEM. While the mapping model predicts the mean or average clearance for each tooth pair, ProTEM also predicts the complete probability distribution of clearances for each pair of teeth. The ranges of predicted clearance values are shown for a 10-tooth spline in Figure 6. Each tooth in the sequence has its own distribution which is different from the normal distribution.

**Objective**

The objective of this paper is to evaluate the Probabilistic Tooth Engagement Model (ProTEM). A Monte Carlo Simulation was used to model several thousand spline assemblies and generate a distribution of the tooth clearance for each tooth in the sequence. The results were compared to the mapping model and to ProTEM.

**Methods**

**Overview of ProTEM**

ProTEM combines three related probabilities for tooth clearance. To calculate the probability of Tooth $j$ on an $N$-tooth spline having clearance $x$, it combines the probabilities that there are:

- $j-1$ teeth with a clearance less than $x$,
- one tooth with a clearance equal to $x$, and
- $N-j$ teeth with a clearance greater than $x$.

Using these probabilities, a probability density function, PDF, is developed for each tooth as a function of $x$. The PDF for the $j^{th}$ tooth to engage on an $N$-tooth spline, $h(x, j, N)$, is defined in Equation 1.
\[ h(x, j, N) = \frac{N!}{(j-1)!(N-j)!} \phi(x)^{j-1} \times (1 - \phi(x))^{N-j} g(x) \]  

(1)

where

- \( g(x) \) is the probability density function for an individual tooth clearance;
- \( \phi(x) \) is the cumulative density function, CDF, for an individual tooth clearance.

Raising \( \phi(x) \) to the power of \( j-1 \) gives the probability that \( j-1 \) teeth have a clearance smaller than \( x \). Likewise, raising \( 1 - \phi(x) \) to the power of \( N-j \) gives the probability that \( N-j \) teeth have a larger clearance value than tooth \( j \)'s clearance, \( x \). The first term in the PDF accounts for equivalent permutations, because tooth \( j \), the \( j^{th} \) tooth to engage, does not have to be in any specific position on the spline.

In this study, the various tooth clearances are assumed to be independent and to be normally distributed. The PDF and CDF for a normal distribution are shown in Figure 7 where the height of the PDF curve at \( x \) is \( g(x) \). The probability of a tooth having a clearance between \( x \) and \( x + dx \) is given by \( g(x) \ dx \). For a normal distribution \( g(x) \) is expressed in equation 2.

\[ g(x) = \frac{1}{\sigma \sqrt{2 \pi}} e^{- \frac{(x-\mu)^2}{2 \sigma^2}} \]  

(2)

**Figure 7.** Manufacturing variation PDF and CDF, as used to determine probabilities in ProTEM
The red area under the PDF to the left of the clearance value, \( x \), gives the probability of a tooth having a clearance smaller than \( x \). The blue area under the PDF to the right of \( x \) gives the probability that a tooth has a clearance larger than \( x \). Calculating these probabilities requires integration of the PDF.

To save repeatedly integrating for each probability value, the CDF, plotted below the PDF curve, evaluates the shaded area under the PDF curve as a function of \( x \). At a given value of \( x \), the height of the CDF curve, \( \Phi(x) \), gives the area integral from \(-\infty\) to \( x \), and represents the probability of a tooth clearance smaller than \( x \). The total area under the PDF is equal to 1.0, which corresponds to the sum of \( \Phi(x) \) and \( 1 - \Phi(x) \), as shown in Figure 7.

Any PDF, whether a normal or other distribution, may be characterized in terms of its area moments. The first moment, or “centroid” of the area under the curve, is the mean value. The second moment, the variance, or the standard deviation squared, describes the range of the distribution. The third moment describes the skewness, or asymmetry. The fourth moment describes the kurtosis, or peakedness. The \( k \)-th moment for the distribution of tooth \( j \) on an N-tooth spline can be found using the moment-generating function, which integrates the product of \( x^k \) and the PDF, \( h(x, j, N) \), as shown in Equation 3 [3].

\[
E\left( x^k \right) = \int_{-\infty}^{\infty} x^k h(x, j, N) \, dx
\]  

(3)

Using this function, the first moment, or mean, is defined in Equation 4, with \( k=1 \).

\[
E(x_j) = \int_{-\infty}^{\infty} \left[ x \left( \frac{N!}{(j-1)!(N-j)!} \right) \phi(x)^{j-1} \times (1 - \phi(x))^{N-j} g(x) \right] \, dx
\]  

(4)

The 2nd, 3rd, and 4th moments are calculated using corresponding \( k \) values. The moment generating function is not closed form, so each of the moments must be calculated numerically.

Because it defines the PDF, ProTEM gives a range of clearance values rather than just a mean value. The PDF for Tooth 1 on a 10-tooth spline is shown in Figure 8. Looking at the PDF, it is evident that the most likely clearance, the peak value, is not the mean clearance; the distribution is skewed to the left (the direction of the longer tail).

**Probability density for clearance of first tooth on 10-tooth spline**

![Figure 8. Tooth clearance predicted by ProTEM for Tooth 1 on a 10-tooth spline](image-url)
In order to generalize the results, ProTEM calculates probabilities for clearances in terms of a standard normal distribution. The results are produced in standard deviation coordinates, meaning that a value of -1.54 corresponds to a clearance value 1.54 standard deviations below the mean clearance. By substituting the manufacturing mean and standard deviation, the results can be scaled to fit each manufactured data set.

In ProTEM, each tooth distribution is skewed away from the global mean clearance, or the manufacturing clearance mean, as shown in Figure 6 for the example of a 10-tooth spline.

Monte Carlo simulation

Monte Carlo Simulation (MCS) was used to verify the ProTEM model. In this simulation, sets of tooth clearances were generated with normally distributed random errors and the resulting distributions for Tooth 1, Tooth 2, etc. were compared to ProTEM.

After generating 100,000 sets of N tooth clearances, each set, representing an individual spline assembly, was sorted in order of increasing clearance to determine the tooth engagement order. The clearance data was then grouped according to the tooth engagement sequence with all the data for tooth j in a set, resulting in N sets of 100,000 clearances each. Each set revealed a skewed distribution, with its mean value shifted to the left or right of the global mean clearance, forming a distribution of distributions, for comparison to the ProTEM solution, as shown in Figure 6.

Distribution from Monte Carlo data

Two different methods were used to generate a CDF from the MCS data. The first method plotted the clearance data as a discrete CDF, from which the PDF was obtained by numerical differentiation. The second method calculated the first four moments of the MCS data, which were fitted to a general skewed distribution.

MCS-Generated Discrete CDF

To generate a discrete CDF from MCS data, it is necessary to assign a percentile (or cumulative probability value) to each data point. This is done by first sorting the data in increasing order. A percentile value is then calculated for point i in a set of N points from equation 5.

\[ P_i = \frac{i - 0.5}{N} \]  

(5)

This formula places the percentile of the smallest clearance in the sample at 1/(2N), and the percentile for the largest clearance in the distribution at 1-1/(2N).

A discrete CDF is then determined by plotting the percentile values \( P_i \) as a function of clearance. This method is shown for 20 sample assemblies in Figure 9. The horizontal axis is in standard deviation units of the global clearance distribution.

![CDF of first tooth for 20 samples](image-url)

Figure 9. CDF for Tooth 1 developed from MCS using 20 sample assemblies
It is clear from this figure that the discrete CDF is not smooth. As more samples are taken, the smoothness will increase. Nevertheless, due to the discrete nature of this distribution, it will never be perfectly smooth.

Figure 10 shows the CDF plotted in the same manner using 100,000 samples. This provides a smoother curve. Also plotted on the same graph is the CDF of a standard normal distribution. From this we can see that the clearance distribution of Tooth 1 is not normal.

In order to generate a PDF, the numerical derivative of the CDF was taken. Initially, a central-difference method was used to take the derivative, as shown in Figure 11. Although the CDF in Figure 10 appears smooth, taking the derivative amplifies the noise that is present. In order to reduce the noise, a five point-moving average was taken to smooth the CDF, and then the PDF was generated by fitting a parabola to a local segment of the CDF and taking the analytical derivative of the parabola at the center of the segment. This creates less noise than a finite difference approach, as shown in Figure 11.

This procedure—creating a CDF from the MCS data, smoothing the curve, and taking the derivative using a parabolic fit—was repeated for each tooth on a 10-tooth spline. The results are shown in Figure 12. The similarity to ProTEM (see Figure 6) is evident: both sets of PDF are skewed away from the overall clearance mean, and the means and spreads appear comparable.

![CDF of first tooth for 100,000 samples](image1)

**Figure 10.** CDF for Tooth 1, developed from MCS using 100,000 samples compared to a normal CDF with the same mean and standard deviation as in the MCS

![PDF of First Tooth for 100,000 Samples](image2)

**Figure 11.** Finite difference approach to taking numerical derivative compared to a parabolic fit
Fitting a distribution to calculated moments

Generating a CDF, smoothing it, and taking the derivative to obtain the PDF, required significant computation, so another method was explored. Rather than directly computing a PDF, an analytical probability distribution was fit to the moments of the distribution. Three distributions were compared: a normal distribution, a generalized gamma distribution, and a lambda distribution.

A normal distribution was fit using the first two moments: the mean and standard deviation. However, it does not include skewness, which is visible in the numerically determined PDF. The generalized gamma distribution uses the first four moments to fit the distribution, but the accuracy of the fit varied based on the overall clearance mean that was chosen. (The mean had to be shifted from zero because the generalized gamma distribution is only valid for x-values greater than zero.)

The lambda distribution was also fit using the first four moments. It provides a close fit when compared to the numerically-generated PDF, as shown for Tooth 3 in Figure 13 and was the best distribution found to model the MCS data [4].

When comparing the MCS distribution to the ProTEM distribution, the lambda fit to the MCS data was used.

Results

Raw data

The mean clearances were found using each method of predicting and simulating. The results are compared for a 10-tooth spline in Figure 14. The MCS results agree almost exactly with ProTEM. The mapping model is close, but the mean value deviates slightly higher for the first tooth and the last tooth to engage. The deviation in the first tooth is the most likely to be significant, because the first tooth is the most highly-loaded tooth on the spline.

The mean clearance of the Tooth 1, the most significant value, was compared between the three methods as the number of teeth on the spline was
increased. The results are shown in Figure 15. MCS and ProTEM agree almost exactly, while the mapping model gives consistently higher results, underestimating the clearance of the first tooth.

The standard deviation of the tooth clearances from ProTEM and MCS are shown in Figure 16. The results agree favorably. The mapping model does not predict the standard deviation.

Table 1 compares numerical results for the first four moments predicted by ProTEM to the moments found through MCS for each tooth on a 10-tooth spline. The results closely match each other. Tooth 1 and Tooth 7 are singled out as typical examples.

Table 2 compares the same moments for the first 10 teeth to engage on a 100-tooth spline. Again, the results compare favorably. Table 3 demonstrates that for the mean clearance, the difference between the ProTEM model and the MCS model is less than 0.5%. For the standard deviation, the difference is less than 1.5%. For the third moment, most of the differences are less than 3%, although one varies by 13% (but it is worth noting that all of the third moments are very small). For the fourth moment, the differences are less than 5%. Given the random nature of the MCS process, these results serve to confirm the validity of the ProTEM model.
Figure 16. Comparison of standard deviation of the clearance for Tooth 1 from ProTEM, and MCS as the number of teeth in the spline is increased.

Table 1. First four moments compared for a 10 tooth spline

<table>
<thead>
<tr>
<th></th>
<th>Monte Carlo Moments*, 100000 samples</th>
<th>Probabilistic Model Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-1.5367</td>
<td>-1.0012</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.34144</td>
<td>0.21557</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>-0.079694</td>
<td>-0.02134</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>0.38557</td>
<td>0.14309</td>
</tr>
</tbody>
</table>

Table 2. First four moments compared for 10 teeth of a 100 tooth spline

<table>
<thead>
<tr>
<th></th>
<th>Monte Carlo Moments*, 100000 samples</th>
<th>Probabilistic Model Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-2.5062</td>
<td>-2.1481</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>0.18499</td>
<td>0.09013</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>-0.052853</td>
<td>-0.012937</td>
</tr>
<tr>
<td>$\mu_4$</td>
<td>0.13149</td>
<td>0.031164</td>
</tr>
</tbody>
</table>

13
Table 3. Quantitative differences of moments between ProTEM and MCS distributions.

<table>
<thead>
<tr>
<th>Moment</th>
<th>10 tooth spline</th>
<th>100 tooth spline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tooth 1</td>
<td>Tooth 7</td>
</tr>
<tr>
<td>μ₁</td>
<td>-0.05%</td>
<td>-0.36%</td>
</tr>
<tr>
<td>μ₂</td>
<td>-1.30%</td>
<td>-0.20%</td>
</tr>
<tr>
<td>μ₃</td>
<td>-2.20%</td>
<td>0.17%</td>
</tr>
<tr>
<td>μ₄</td>
<td>-3.43%</td>
<td>0.16%</td>
</tr>
</tbody>
</table>

Distributions compared

In addition to comparing the raw data, the distributions were compared. (The mapping model does not produce a distribution of tooth clearance, and is not shown here.) Figure 17 shows graphical comparisons of ProTEM to MCS for a 10-tooth spline, and for the first 10 teeth to engage on a 20-, 40-, and 100-tooth spline. The MCS plots shown are the lambda fits to the MCS data (which were determined to be equivalent to the numerically generated MCS distributions). The plots correlate very closely for a 10-tooth and 20-tooth spline. The distributions appear to diverge more for a 40-tooth and 100-tooth spline; however, they still show a very similar range of probable clearances for each tooth. Additionally, the first tooth clearance, which is the most critical in predicting tooth failure, is still very closely correlated between ProTEM and MCS.

Conclusion

The results from ProTEM compare favorably both quantitatively (looking at the moments) and qualitatively (comparing the distributions) with MCS using a large number of samples. This comparison validates ProTEM as a predictor of probable tooth clearances.

Methods compared

MCS and ProTEM give equivalent results for clearance variation in splines; however, ProTEM is more efficient to use. Table 4 compares the tasks involved in generating the data of interest. It is clear from this table that ProTEM does not require as many tasks or as much computation to gather the desired information about the clearances as MCS does.
Table 5 compares the accuracy, necessary computation, and the process of determining quality level for each method. From this we see that ProTEM requires the least computation, while still providing high accuracy. Determining the quality level in production requires less computation using ProTEM than using either approach with MCS.

An important difference between MCS and ProTEM is that MCS requires the whole spline to be modeled, using several thousand sample assemblies, to be able to gather pertinent information, while ProTEM analyzes only the first few teeth in a single assembly.

Applications

This study is a continuation of a statistical investigation of involute spline performance. The previous work developed a strength of materials model for tooth stiffness, which is an essential element in predicting tooth engagement and load sharing. This model was verified by finite element analysis. The results were used to develop a spreadsheet (called STEM), based on the mapping concept, to statistically predict the most likely, or average tooth engagement.

ProTEM is a more sophisticated statistical model. It provides a tool to predict the range of tooth clearances statistically for each tooth on an N-tooth spline. This information may be used to determine the probable values (mean value and range about the mean) for the following:

- Sequence of tooth engagement
- Number of teeth carrying a specified load
- Load distribution among teeth
- Loads and stresses in each tooth

By having this information, engineers will have an increased understanding of the mechanics of spline engagement. Designers are provided with a tool to accurately predict performance of splines. This has the potential to be used to optimize the design and produce higher performance splines.

### Table 4. Comparison of computational efficiency

<table>
<thead>
<tr>
<th>Tasks involved in generating probability distribution and finding mean and range</th>
<th>MCS: Direct numerical computation</th>
<th>MCS: Lambda fit to data</th>
<th>ProTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perform simulation, generating over a million values used in computation</td>
<td>• Perform simulation, generating over a million values used in computation</td>
<td>• Get distribution directly from formula, for the given tooth number and number of teeth on spline</td>
<td></td>
</tr>
<tr>
<td>Compute mean and range directly from data</td>
<td>• Compute mean, range, and other moments directly from data</td>
<td>• Calculate range from distribution</td>
<td></td>
</tr>
<tr>
<td>Plot and smooth CDF</td>
<td>• Use moments to fit Lambda distribution</td>
<td>• Calculate mean from moment-generating function</td>
<td></td>
</tr>
<tr>
<td>Fit parabola segments using least squares</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Take numerical derivative to get distribution</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5. Comparison of accuracy, computation, and accessibility

<table>
<thead>
<tr>
<th>Method</th>
<th>MCS: Direct numerical computation</th>
<th>MCS: Lambda fit to data</th>
<th>ProTEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy of method</td>
<td>Dependent on sample size: requires more than 10,000 samples, generating a complete distribution for each tooth</td>
<td>Good: agrees well with MCS. Equivalent to a large sample size</td>
<td></td>
</tr>
<tr>
<td>Computation required</td>
<td>Most computation</td>
<td>Much computation</td>
<td>Least computation</td>
</tr>
<tr>
<td>Process of determining quality level</td>
<td>Perform entire simulation; count the number of splines that fail to meet design requirements</td>
<td>Perform entire simulation; use tabulated lambda data to predict how many splines fail to meet design requirements</td>
<td>Integrate PDF of Tooth 1 to predict how many splines fail to meet design requirements</td>
</tr>
</tbody>
</table>
Future work

As a result of this study, our sponsor was motivated to test a spline assembly under load, to measure the torque vs. rotation accurately and see if the predicted piece-wise linear behavior, shown in Figure 2, could be verified. The results were highly successful. They even demonstrated that disassembling the spline and reassembling it, rotated to a new position, produced a completely new torque-rotation plot, as predicted, since all the tooth clearances would change. These results are presented by DeCaires [1].

One surprising feature was discovered. Some of the increments in slope were much larger than predicted. Examination revealed that they appeared to be integer multiples of the single tooth increment expected, indicating that more than one tooth was engaging simultaneously. This behavior was not predicted by the tooth engagement model.

It has been postulated that perhaps there is a periodic fluctuation in the hobbing or generating process that leads to correlation between teeth, in violation of the assumption of independence. Perhaps it is a numeric interaction between the number of teeth on the spline and the number of teeth, or threads, on the cutter.

This opens up new possibilities. If the correlation mechanism can be found, it may be possible to design the process to deliberately cause the teeth to engage in groups favorable to load sharing. It could lead to splines with higher load carrying capabilities.

A third research effort has recently been conducted to analyze measured spline data provided by our sponsor. The data was of sufficient resolution to calculate the expected clearances between mating spline teeth and to predict the tooth engagement sequence. It also enabled demonstration of the changes which occur when a spline is disassembled, rotated and reassembled. Additionally, a preliminary study was made to simulate, by means of a CAD model, how errors in the hobbing process are imprinted on production splines.

Effective tooth engagement

When determining the tooth loading, it is common design practice to divide the total applied load by the number of teeth assumed to be engaged. Typical recommended values are ¼ or ½ of the total teeth, depending on how heavy the load. However, even if the number of teeth engaged were known, this underestimates the maximum tooth load, since the load is not distributed uniformly. As shown in Figure 18, Tooth 1 carries 18% of the total load when 10 teeth are engaged. Common design practice would assume uniform loading on the engaged teeth, where each tooth carries 10% of the load. This assumes a lower force on Tooth 1 than is actually present, and the tooth will fail before a uniform loading would predict.

Figure 18. Effective tooth engagement and the percent of load carried by 10 engaged teeth [1]
In order to remedy this, the total load can be divided by an effective number of teeth engaged. The effective number of teeth engaged is determined from the Statistical Tooth Engagement Model (STEM), by dividing the total load by the amount carried by Tooth 1, which is estimated by modeling probable clearances of a spline. The STEM spreadsheet was developed for this purpose [1]. In the case of Figure 18, it would take five teeth, at a load equal to Tooth 1, to carry the load. Dividing the load by the effective number of teeth engaged gives an accurate estimate of the maximum tooth load, which can then be used to determine at what load the spline will fail.

The Effective Tooth Engagement provides a design procedure similar to that which engineers commonly use, but which yields a much more accurate estimate of tooth loads and stress.

References